

# How to Support an Unknown Future: Preprocessing for Local Algorithms

Klaus-Tycho Foerster, Juho Hirvonen, Stefan Schmid, and Jukka Suomela. To appear @IEEE INFOCOM 2019









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- Cannot improve in the LOCAL model  $\ensuremath{\mathfrak{S}}$





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- What are further application scenarios?
- What else can we do with the SUPPORT of Preprocessing?





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   Create many local controllers that can react quickly, that control small set of "dumb" nodes



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12/10/2018 Preprocessing for Local Algorithms. Talk @ETH Zurich, Distributed Computing Group. Host: Roger Wattenhofer



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Active variant: allow to

communicate on support H

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  - Not even for the *active* variant





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In *active* model:

**Congested Clique** 

Idea: simulate that support graph H is a

complete graph



## But: Restricted Graph Families are Useful ③

- Real topologies are usually not complete graphs
- Case study: planar graphs
  - Remain planar under edge deletions
  - Are 4-colorable



"Geloeste und ungeloeste Mathematische Probleme aus alter und neuer Zeit" by Heinrich Tietze http://www.math.harvard.edu/~knill/graphgeometry/faqg.html





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  SUPPORTED speed-up:

  precompute 4-coloring
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  reduce 4-colored pseudo-forest to 3 colors in 2 rounds

  Find weight-appropriate pseudo-forest [non-constant time 🔅]
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  - Also for planar graphs for maximum independent set & maximum matching











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 $2^{O(\sqrt{\log n})}$




Also works in *passive* model: SLOCAL(t)  $\rightarrow$ SUPPORTED( $\Delta^{O(t)}$ )

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## Further Results in the Active SUPPORTED Model

- Connection to SLOCAL model [Ghaffari et al., STOC 2017]
  - SLOCAL(t) can be simulated in SUPORTED(O(t\*poly log n)): e.g. MIS in SUPPORTED(poly log n)

Use all edges of H

for communication

- Converse not true, respectively open question







• LCL in LOCAL(o(log n)) can be solved in O(1) in the SUPPORTED model





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  - Set of size  $(\alpha(G)-\epsilon)n$  in  $O(\log_{1+\epsilon} n)$ , respectively  $(1+\epsilon)$  approximation if maximum degree  $\Delta$  constant



- Optimization problem: Maximum Independent Set, of size  $\alpha(G)$ 
  - $\circ$  Set of size (α(G)-ε)n in O(log<sub>1+ε</sub> n), respectively (1+ε) approximation if maximum degree Δ constant
  - $\circ$  Cannot be approximated by  $o(\Delta/\log \Delta)$  in time  $o(\log_{\Delta} n)$  in the active SUPPORTED model



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